Fuzzy Logic Controller based Pitch Control of Aircraft tuned with Bees Algorithm

R. Zaeri, A. Ghanbarzadeh, B. Attaran

Abstract—For linear systems and some of non-severe non-linear systems, classic controllers such as PI and PID have been widely used in industrial control processes because of their simple structure and robust performance in a wide range of operating conditions. Several numerical approaches such as Fuzzy Logic Controller (FLC) algorithm and evolutionary algorithms have been used for the optimum design of PID controllers. In this paper, a pitch displacement of aircraft was controlled by FLC tuned with Bees Algorithm (BA). For a given input, the parameters of Mamdani-type Fuzzy Logic Controller (the centers and the widths of the triangle membership functions (MFs) in inputs and output) were optimized by the BA with Integral Time Absolute Error (ITAE) as a cost function. In order to compare the optimized Fuzzy Logic Controller with different controller, the PI controller was tuned with BA and also PI controller tuned with Ziegler-Nichols tuning rules. The simulation results show that Fuzzy Logic Controller tuned by bees algorithm is better performance and more robust than the fuzzy-Expert and PI tuned by bees algorithm and Ziegler-Nichols for aircraft pitch control.

I. INTRODUCTION

FLC is popular technique that has seen increasing interest in the past decades since it has a linguistic based structure and its performance are quite robust for controlling systems. However, FLC including some parameters such as linguistic control rules and limits and type of MFs have to be tuned for a given system. A major drawback of FLC is that the tuning process becomes more difficult and very time consuming when the number of the system inputs and outputs are increased. Bees algorithm (BA) is relatively new evolutionary algorithm that may be used to find optimal or near optimal solutions in big search space [6]. BA is especially useful for parameter optimization in continuous, multi-dimensional, search spaces and multi-objective, complex optimization [7], [8]. The BA method can generate a high quality solution within shorter calculation time and it tends to converge very fast compared to other stochastic methods. The BA has proven both very effective and quick in diverse set of benchmark optimization problems [9]-[12]. Evolutionary algorithms regarding tuning the MFs parameters of FLC have been studied extensively in the literature. These studies can be divided into four groups as genetic algorithm, particle swarm optimization (PSO), ant colony optimization (ACO) and bees algorithm (BA) [13]-[20]. Attaran and ghanbarzadeh optimized FLC MFs for hydraulic actuator of gun turret using BA [9]. Also BA was used to Fuzzy clustering [11]. Pham et al. tuned a fuzzy logic controller for a robot gymnast using the BA [10]. Myint et al. designed a PID controller for pitch stability of Piper Cherokee aircraft. Their results showed that the PI controller based Ziegler-Nichols tuning rules has better performance respect to PID [21]. Kurnaz et al. proposed an ANFIS (adaptive neuro-fuzzy inference system) based controllers for unmanned aerial vehicles (UAVs). They used MATLAB’s standard configuration to control the position of the UAV in three dimensional space as altitude and longitude–latitude location [22].

In this work, BA was used to tune the Mamdani type-fuzzy controller’s antecedent and consequent parameters for the pitch attitude control system. The rest of this article is organized as follows. The components of a pitch attitude control system of the piper aircraft are described as a case study and also the classic PI controller which tuned with Ziegler-Nichols tuning rules was designed in Section 2. A fuzzy control algorithm is proposed in Section 3. The BA tuning method for the fuzzy controller and the PI controller is described in Section 4. The results and conclusion are given in Sections 5 and 6, respectively. Results show that the performance of combining Fuzzy-BA is considerably improved.

II. AIRCRAFT CONTROL

A. Aircraft aerodynamics

Aircraft fly in three-axis plane by controlling aileron, rudder, elevator, and throttle. The control system of the aircraft is divided into two portions; longitudinal and lateral
control as shown Fig. 1. To control the pitch displacement, elevators are used. These are usually situated at the rear of the aircraft running parallel to the wings that house the ailerons. In order to raise the pitch of the craft (front upwards, rear downwards) the elevators are lifted causing the rotation of the craft about the axis through the middle of the craft from left to right as shown Fig. 2.

**A. Case study: Piper Cherokee Aircraft**

The equations are nonlinear and are linearised to facilitate their solution by assuming the motion of the aircraft to be small perturbations about a trim condition. The equations for the longitudinal motion contain coefficients $X_u$, $X_w$ etc., which are called aerodynamic stability derivatives where the symbol $\frac{\partial}{\partial}$ denotes the derivatives to be dimensional. For example, $X_u$ represents the change in the force due to perturbation in the velocity in the $x$ direction $u$ [23].

\[
\begin{align*}
\dot{m}u - \dot{X}_u & = X_u w - \dot{X}_w w - \dot{X}_\theta \theta + mg \theta = \dot{X}_\theta \eta + \dot{X}_\theta \tau \\
-\dot{Z}_u + (m - \dot{Z}_w) & = \dot{Z}_w w - (\dot{Z}_\theta + mU \theta) \theta = \dot{Z}_\theta \eta + \dot{Z}_\theta \tau \\
-\dot{M}_u - M_u w - M_w w + I, \dot{\theta} - M_w \theta = M_u \eta + M_w \tau
\end{align*}
\]

(1)

Using aircraft data and coefficients, the piper Cherokee aircraft flying in straight and level flight with weight 2400lb at attitude 10000ft of mach 0.1907 at airspeed 140mph, have been calculated for pitching transfer functions from Eq. (1) [24]. The components of a pitch attitude control system are shown in Fig. 3. For this design the reference pitch angle is compared with the actual angle measured by a gyro to produce an error signal to activate the control servo. In general, the error signal is amplified and sent to the control surface causes the aircraft to achieve a new pitch orientation, which is fed back to close the loop. The elevator servo transfer function represented as a first-order system in Eq. (3).

\[
G_e(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{17.1338(s^2 + 1.6557s + 0.0577)}{s^2 + 5.4943s^3 + 36.1149s^2 + 1.0357s + 1.4186}
\]

(2)

\[
G_e(s) = \frac{\delta_e}{v} = \frac{k_e}{\tau s + 1}
\]

(3)

Where $\delta_e$, $v$, $k_e$, and $\tau$ are the elevator deflection angle, input voltage, elevator serve gain and servomotor time constant respectively. Time constant for typical servomotors falls in a range of 0.05-0.25sec. In this design, assume time constant is 0.1sec and $k_e$ is -1.

Finally the transfer function must be controlled with PI represented with Eq. 4.

\[
\begin{align*}
\theta(s) & = 171.338(s^2 + 1.6557s + 0.0577) \\
\delta_e(s) & = \frac{15.4943s^2 + 262.395s^3 + 145.869s^2 + 21.6618s + 14.186}{s^2 + 15.4943s^3 + 262.395s^3 + 645.869s^2 + 21.6618s + 14.186}
\end{align*}
\]

(4)

**B. Classical Controller**

To tune the PI controller (in this study because of zero existence in system and sensitivity of derivative to zero, PI controller is the best choice for the system to be stable) the Ziegler-Nichols method which is frequently used in industrial applications designed from the plant step response [25]. Using Routh criterion, $K_{cr}=18.044$ and $P_{cr}=18.044$ were obtained. Table I shown the PI gain coefficients.
### TABLE I. ZIEGLER-NICHOLS PI GAINS

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>P&lt;sub&gt;c&lt;/sub&gt;</th>
<th>K&lt;sub&gt;p&lt;/sub&gt;</th>
<th>K&lt;sub&gt;i&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>2πω</td>
<td>0.45K&lt;sub&gt;p&lt;/sub&gt;</td>
<td>0.45K&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>0.4054</td>
<td>8.1216</td>
<td>24.1368</td>
</tr>
</tbody>
</table>

The PI controller was derived from Eq. (5).

\[
G_{s,p}(s) = K_p + \frac{K_i}{s} \tag{5}
\]

### III. FUZZY LOGIC CONTROLLER

The FLC has three main components such as fuzzification, fuzzy inference engine (decision logic), and defuzzification stages. The block diagram of FLC is shown in Fig. 4.

The first block in the Fig. 4 is fuzzification which converts each element of input data to degrees of membership by a lookup in one or several MFs. The rule base and inference base have the capability of simulating human decision-making (Fuzzy-Expert) based on fuzzy concepts and the capability of inferring fuzzy control actions employing fuzzy implication and the rules of inference in fuzzy logic. The MFs of the fuzzy sets and the fuzzy control rules have a big effect on control performance [21], [26]-[27]. The third operation is called as defuzzification. The resulting fuzzy set is defuzzified into a crisp control signal. There are five defuzzification methods: centroid, bisector, middle of maximum, largest of maximum, and smallest of maximum [27].

In the aircraft pitch control here, the input variables of the FLC are error, \( e \) (deg) and output derivation, \( \dot{y} \) (deg/s) of aircraft pitch angle. The values of the \( e \), \( \dot{y} \) and output are scaled to the interval of [-1 1], [-6.5 6.5] and [-10 10]. \( e \) and \( \dot{y} \) are mapped to linguistic variables error and \( \dot{y} \) by the fuzzification operator. The FLC inputs are composed of the seven linguistic terms NB (Negative Big), NS (Negative Small), NZ (Negative Near Zero), Z (Zero), PZ (Positive Near Zero), PS (Positive Small) and PB (Positive Big). Also, the pitch angle for the aircraft (FLC outputs) is partitioned into the same seven fuzzy set. This set of linguistic terms forms a fuzzy partition of input and output spaces. Fig. 5 shows the initial MFs of the fuzzy sets for the angle error, changes in the output angle, and control output and their fuzzy control surface.

The Triangle MFs were used for both input and output of FLC. Triangle membership function is defined as Eq. (6).

\[
f(x,a,b,c) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & c \leq x 
\end{cases} \tag{6}
\]

where \( a \) and \( c \) locate the "feet" and the parameter \( b \) locates the peak of the triangle membership function. The fuzzy if-then rules for pitch control are given in Table II. The total number of rules is 49 in Table II.

### TABLE II. FUZZY LOGIC RULE BASE

<table>
<thead>
<tr>
<th>ydot/error</th>
<th>NB</th>
<th>NS</th>
<th>NZ</th>
<th>Z</th>
<th>PZ</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>Z</td>
<td>PZ</td>
<td>PS</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
<tr>
<td>NS</td>
<td>NZ</td>
<td>Z</td>
<td>PZ</td>
<td>PS</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
<tr>
<td>NZ</td>
<td>NS</td>
<td>NZ</td>
<td>Z</td>
<td>PZ</td>
<td>PS</td>
<td>PS</td>
<td>PB</td>
</tr>
<tr>
<td>Z</td>
<td>NB</td>
<td>NS</td>
<td>NZ</td>
<td>Z</td>
<td>PZ</td>
<td>PS</td>
<td>PB</td>
</tr>
<tr>
<td>PZ</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>Z</td>
<td>PZ</td>
<td>PS</td>
</tr>
<tr>
<td>PS</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>NZ</td>
<td>Z</td>
<td>PZ</td>
</tr>
<tr>
<td>PB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>NZ</td>
<td>Z</td>
<td>Z</td>
</tr>
</tbody>
</table>

The resulting fuzzy set must be converted to a signal that can be sent to the process as a control input. Bisector of area was used here for defuzzification schema. The simulink model of FLC for the aircraft is shown in Fig. 6.

### IV. BEES ALGORITHM

#### A. The Bees Algorithm

The Bees Algorithm is an optimization algorithm inspired by the natural foraging behavior of honey bees [6]. Table III shows the pseudo code for the algorithm in its simplest form.

### TABLE III. PSEUDO CODE FOR BA

1. Initialize population with random solutions.
2. Evaluate fitness of the population.
3. While (stopping criterion not met) // Forming new population.
4. Select elite bees and elite sites for neighborhood search.
5. Select other sites for neighborhood search.
6. Recruit bees around selected sites and evaluate their fitness.
7. Select the fittest bee from each site.
8. Assign the remaining bees to search randomly and evaluate their fitness.
9. End while.
The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of sites selected out of n visited sites (m), number of elite sites out of m selected sites (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other (m-e) selected sites (nsp), initial size of patches (ngh) which includes site and its neighborhood and stopping criterion. The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2. The valuation of fitness would depend on the optimization problem, but in general ‘fitness’ is taken as the value of the objective function being optimized. In step 4, bees that have the highest fitnesses are designated as “selected bees” and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best e sites. The bees can be chosen directly according to the fitnesses associated with the sites they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best e sites which represent more promising solutions are made more detailed by recruiting more bees to follow selected bees than the other selected bees. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm. However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This constraint is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each Iteration, the colony will have two parts to its new population representatives from each selected patch and other scout bees assigned to conduct random searches. The Bees Algorithm was adopted in this work as it had proved to have a more robust performance than other intelligent optimization methods for a range of complex problems [6].

The values of the parameters of the Bees Algorithm for FLC are shown in Table IV.
TABLE IV. THE BEES ALGORITHM PARAMETERS FOR FLC

<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>e</th>
<th>nsp</th>
<th>nep</th>
<th>ngh</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>15</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**C. Optimization of PI with BA**

Tuning process with BA using ITAE (Eq. (7)) cost function is applied to the PI controller in order to show the differences between the PI-Ziegler-Nichols, Fuzzy-Expert, Fuzzy-BA and the PI-BA controllers and compare their performances.

The optimization conditions for both the FLC and the PI-BA controller should be same for accurate comparison. The PI constants optimized with ITAE cost function using BA and classic PI constants are summarized in Table IV.

![Fig. 7. Optimized MFs for (a) pitch angle error, (b) output derivation, (c) output, (d) control surface](image)

TABLE V. PI COSTANTS OF ZIEGLER-NICHOLS AND PI-BA

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_p$</th>
<th>$K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI-Ziegler-Nichols</td>
<td>8.1216</td>
<td>24.1368</td>
</tr>
<tr>
<td>PI-BA</td>
<td>8.3517</td>
<td>1.5284</td>
</tr>
</tbody>
</table>

The values of the parameters of the Bees Algorithm for PI are shown in Table VI.

TABLE VI. THE BEES ALGORITHM PARAMETERS FOR PI

<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>e</th>
<th>nsp</th>
<th>nep</th>
<th>ngh</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**V. RESULTS**

The objective of this section is to test the performance of the Fuzzy-BA controller in comparison with FLC-Expert, PI-BA and classic PI controllers by minimizing the settling time, percent of overshoot and hence improve the dynamic stability of the pitch displacement of aircraft.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Max overshoot (%)</th>
<th>Settling Time 2% (sec)</th>
<th>Settling Time 5% (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI-Ziegler-Nichols</td>
<td>40</td>
<td>1.47</td>
<td>1.35</td>
</tr>
<tr>
<td>PI-BA</td>
<td>20.19</td>
<td>3.04</td>
<td>1.05</td>
</tr>
<tr>
<td>Fuzzy-Expert</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Fuzzy-BA</td>
<td>6.22</td>
<td>0.41</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The Fuzzy-BA controller performs better than the PI-BA controller in terms of the convergence and steady state error. The results are shown in Fig. 8. Also for more realization, results were presented in Table VII.

![Fig. 8. Step Response for (a) FLC tuned with Expert, (b) PI tuned with Ziegler-Nichols, (c) PI tuned with BA, (d) FLC tuned with BA](image)
As can be seen, the performance of the Fuzzy-BA is better compared to the PI-BA and Fuzzy-Expert controllers. The tabulated results of performance analysis show that the Fuzzy-BA controller has shorter settling time and fast response time than the other controllers. The percent overshoot minimized to negligible percents in Fuzzy and its zero in Fuzzy-BA. For PI-BA controller, the percent of overshoot is negligible but the response settling time and the steady state error cannot be minimized for the reason that Integral gain effect cannot be provided. Also Fuzzy-Expert could not satisfy the input condition and in long simulation, was not able to reach to final value of input.

VI. CONCLUSION

This paper introduced the BA based tuning method for FLC and PI controller to control the given pitch displacement. The all parameters concerning the fuzzy controller and the PI controller were determined using BA algorithm. As a result, The Fuzzy-BA controller can achieve better accuracy and has less or no deviation from the step input compared to the PI-BA controller and another controllers which was studied. It is verified that the Fuzzy-BA controller has better control performance in pitch displacement control. Furthermore, implementation of the FLC tuning with BA is much easier than the traditional methods because there is need neither derivative knowledge nor complex mathematical equations.

References