Hydraulic Jump in a Gradually Expanding Channel with Different Divergence Angles

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ABSTRACT

This paper describes the hydraulic jump characteristics in a gradually expanding channel. The experiments were carried out in a flume with 12 m length, 0.25 and 0.5 m width and depth respectively. Five physical models were built based on the divergence angles, including 5, 12.5, 15, 22.5 and 25 degrees and were fixed in the flume. Jump was controlled to be occurred along the structure. The measured characteristics of the hydraulic jump for different discharges and Froude numbers were compared with each other and with the corresponding values measured for the classical jump. The results showed that the relative sequent depths and the relative length of the jump decrease, whereas the relative energy loss increases with increasing the expansion angle. The maximum relative energy loss was obtained for the angle of 25° and it was about 55% more than the relative energy loss for the classical jump with a Froude number of 4.

Keywords: Hydraulic jump, expanding channel, stilling basin, transitions

INTRODUCTION

At the downstream of many hydraulic structures such as spillways, chutes, drops, gates and etc., the supper critical flow must be controlled to protect the channel and related structures from erosion and possible destruction. One of the most popular energy dissipater in hydraulic networks is stilling basin with hydraulic jump. In this type of structures the excess energy is dissipated using hydraulic jump, with their efficiency being very important from the engineering viewpoint. It is obvious the dimensions of the basin is an important economical factor, with the sequent depths and length in a hydraulic jump being the prevalent items for designing of stilling basin dimensions. There are plenty of research studies in the literature regarding the classical hydraulic jump in the usual rectangular straight stilling basin, but less for the hydraulic jump in other cross section shape of basins. Expanding gradually basin with the rectangular cross section acts as two separate hydraulic structures including stilling basin and transition. In this type of structures not only the transition can be eliminated, but
the length of the basin will be also much smaller than what is designed for the usual straight basins.

There are a few research studies in the literature representing the hydraulic jump properties in either non-rectangular or gradually expanding basins. Posey and Hasing in 1938 (see Omid et al., 2007) investigated the effect of side channel slope in trapezoidal cross section on the length of hydraulic jump. A theoretical equation for the relative sequent depths of hydraulic jump in a trapezoidal cross section was derived by Diskin (1961). Arabhabahirama and Abella (1971) investigated the properties of hydraulic jump in rectangular expansion channel. They assumed a quarter of an ellipse for the water surface profile along the length of the jump and by applying the continuity and momentum equations developed some relationships to calculate the sequent depths and relative energy loss. They verified their equations using some laboratory experiments on a rectangular expansion channel with a maximum diversion angle of 13°. Wanoschek and Hagar (1989) investigated the properties of hydraulic jump in a trapezoidal channel with side slope of 1:1 using laboratory experiments. They found that the sequent depth decreases and the energy loss increases when compared with the usual hydraulic jump in rectangular basin. Omid et al. (2005) modeled the sequent depth and the length of jump in gradually expanding channels having rectangular or trapezoidal cross sections using artificial neural networks. Yan et al. (2006) statistically analyzed the pressure fluctuations at the bottom of spatial hydraulic jumps with abrupt lateral expansions. They recorded the pressure data for different Froude numbers ranging 3.52 to 6.86 and channel expansion ratios ranging from 1.5 to 3.0. They found that the peak frequencies and intensity coefficients of pressure fluctuations are higher than those of the corresponding classical jumps. The most recent research study regarding some properties of hydraulic jump in expanding channels with trapezoidal cross sections has been carried out by Omid et al. (2007). The divergent angle in their study was ranged from 3 to 9°. They found that the sequent depth and the length of jump decrease, whereas the energy loss increases with increasing bottom width.

In this research study the properties of hydraulic jump such as, sequent depth, the length of jump and energy loss is investigated in gradually expanding rectangular channels with a wide range of divergent angles, with the results being compared with the corresponding values for classical jump.

THEORY OF HYDRAULIC JUMP IN EXPANDING CHANNELS

Momentum and continuity equations are usually applied to calculate the sequent depth and energy equation can be then used to specify the energy loss due to hydraulic jump. The main difference between the classical jump and the jump in expanding channels is the lateral forces due to divergent angle. The amount of these forces depends on the length of the jump, and thus the water surface profile is necessary for calculating them.

The hydraulic jump analysis in expanding channels is based on a few assumptions including: the streamlines are radial, flow is steady, uncompressible and uniform at the upstream of jump. Also the pressure everywhere is hydrostatic. Figure 1 shows a definition sketch of hydraulic jump in an expanding channel. According to Figure 1, the momentum equation can be written as (Arabhaabhirama and Abela, 1971):
\[ F_{x1} - F_{x2} + F_{sx} = \int \int \rho \overline{V} d\overline{A} \]  

(1)

where \( F_{x1}, F_{x2}, \) and \( F_{sx} \) are the forces due to hydrostatic pressure at the cross sections 1 and 2, and the \( x \)- component of the lateral force due to divergent angle, respectively. It is easily shown that:

\[ F_{x1} = \frac{\gamma y_1^2 r_1}{2} \sin \theta, \quad F_{x2} = \frac{\gamma y_2^2 r_2}{2} \sin \theta \]  

(2)

where \( r_1 \) and \( r_2 \) are the radial distances from the center of jump at the points 1 and 2 (Figure 1). The lateral pressure is assumed to be hydrostatic and thus \( F_{sx} \) can be written as:

\[ F_s = \int r \frac{\gamma y^2}{2} dx \]  

(3)

where \( L_j \) is the jump length. The water surface profile along the jump is assumed to be quarter-elliptical with the horizontal major semi-axis being equal to \( L_j \).

![Figure 1: Jump configuration in an expanding channel](image)

Therefore, if \( y_1 \) and \( y_2 \) be the sequent depths, the equation of surface profile may be written as:

\[
\frac{(y - y_1)^2}{(y_2 - y_1)^2} + \frac{(x - L_j)^2}{L_j^2} = 1
\]  

(4)

By using Equations 3 and 4 the \( x \) component of \( F_s \) is written as:

\[ F_{sx} = \gamma (r_2 - r_1) \left( \frac{y_2^2}{3} + 0.118 y_2 y_1 + 0.048 y_1^2 \right) \sin \theta \]  

(5)

The right hand sight of Equation 1 is equal to

\[ \int \int \rho \overline{V} d\overline{A} = \left( \rho V_2^2 y_2 r_2 - \rho V_1^2 y_1 r_1 \right) \sin \theta \]  

(6)

By substituting Equations 6, 5, and 2 in Equation 1 and considering continuity equation the dimensionless jump equation in gradually expanding channels will be derived as:

\[ 2\left( r_0 - 1 \right) \left( \frac{y_0^2}{3} + 0.118 y_0 + 0.048 \right) + 1 - r_0 y_0^2 = 2F_1^2 \left( \frac{1 - r_0 y_0}{r_0 y_0} \right) \]  

(7)

where, \( r_0 = \frac{r_2}{r_1}, y_0 = \frac{y_2}{y_1}, \) and \( F_1^2 = \frac{V_1^2}{g y_1}. \)
The energy loss \( E_j \) due to hydraulic jump in an expanding channel is computed using the following equation:

\[
E_j = \frac{F_i^2}{y_i} \left( \frac{r_0^2 y_0^2 - 1}{r_0^2 y_0^2} \right) + (1 - y_0) \tag{8}
\]

For the classical jump in rectangular cross section the relative sequent depths and the relative energy loss based on \( F_i \) is written as (Subramanya, 1991):

\[
\left( \frac{y_2}{y_1} \right) = 0.5 \sqrt{1 + 8 F_i^2 - 1} \tag{9}
\]

\[
\frac{E_j}{E_i} = \frac{-3 + \sqrt{1 + 8 F_i^2}}{8(2 + F_i^2)(1 + \sqrt{1 + 8 F_i^2})} \tag{10}
\]

**EXPERIMENTAL SET UP**

The experiments were performed in a re-circulating tilting flume having a length of 12m, width of 0.25m and depth of 0.5m. Bed slope of the flume was set to zero and the discharges were measured using a standard triangular weir, which was installed in a box at the end of the flume. For creating supper critical flows with different Froude numbers a gate was installed in the middle of the flume and for any case the jump was controlled by another gate at the end of the flume to make sure of occurrence of a free jump in the expanding channel.

To simulate the expanding channel, five physical models were built from glass with divergence angles of 5, 12.5, 15, 22.5 and 25.5°. For a set of experiments each model was tightly installed immediately after the gate. The depth of flow was measured using a point gauge with a reading accuracy of \( \pm 0.1 \) mm and the length of jump was measured using a ruler with an accuracy of 1mm. The measured hydraulic parameters during each experiment were width of the rectangular channel at points 1 and 2 \( (B_1, B_2) \), sequent depths \( (y_1, y_2) \), \( L_j \) and discharge. It should be added that for all experiments the jumps were free. The experimental ranges of the hydraulic parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream depth, ( y_1 ) (mm)</td>
<td>139.4</td>
<td>27.6</td>
</tr>
<tr>
<td>Downstream depth, ( y_2 ) (mm)</td>
<td>352.4</td>
<td>87.2</td>
</tr>
<tr>
<td>Froude number, ( F_i )</td>
<td>6.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Length of jump, ( L_j ) (cm)</td>
<td>71.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Discharge, ( Q ) (lit/s)</td>
<td>50.2</td>
<td>9.5</td>
</tr>
</tbody>
</table>

**RESULTS AND DISCUSSION**

About 75 sets of measured data were collected in expanding part of the flume with different divergence angles. The water surface profile for all tests was experimentally provided to show that the quarter-elliptical assumption for the profile shape is almost valid. Figure 1, as an example, reasonably compares the
measured and theoretical water surface profile for the test in which \( F_1 = 4.39 \) and the divergence angle of \( \theta = 5^\circ \).

The relative sequent depths \( \left( \frac{y_2}{y_1} \right) \) for different divergence angles and Froude numbers were calculated using the measured values, and are compared with the corresponding theoretical values obtained for the classical jumps in a rectangular cross section (Equation 9) in Figures 2(a) to 2(f). As can be seen from these figures, \( \frac{y_2}{y_1} \) is generally decreased with increasing the divergence angles. This reduction was increased by increasing Froude numbers. It should be noted that producing very high Froude numbers in the flume was difficult. According to the results in Figure 2(f) the sequent depth ratio, \( \left( \frac{y_2}{y_1} \right) \) was calculated for \( F_1 = 2 \) and 4 as \((2.37, 2.21, 2.04, 1.90, 1.80, 1.75)\) and \((5.18, 4.32, 4.18, 4.02, 3.70, 3.69)\) for classical and for the divergence angles of 5, 12.5, 15, 22.5 and 25 degrees respectively. The maximum observed reduction for this ratio was calculated 26.2% and 28.8% for Froude numbers of 2 and 4 respectively.

![Figure 1: Comparison of the measured and theoretical water surface profile](image)

The second hydraulic characteristic of the jump considered in this research study was energy loss. The measured relative energy losses \( \left( \frac{E_j}{E_t} \right) \) due to the jump were calculated for different divergence angles and were compared with the theoretical energy loss for classical jump (Equation 10) in Figures 3(a) to 3(f). As can be seen from these figures for the jump in all expanding channels with different divergence angles the relative energy loss is greater than the corresponding one in rectangular cross section. This is due to the lateral forces and more turbulent and rolling flow along the longitudinal section. The trend of the measured values for each channel in these figures shows that for the same Froude number the relative energy loss is increased with increasing divergence angles. The results for divergence angles of 22.5 and 25 degrees were almost similar. According to the trends of the measured values for example, for Froude number equal 4 the relative energy loss was calculated as 0.38, 0.44, 0.53, 0.54, 0.58 and 0.59 for the rectangular and expanding channels with the divergence angles of 5, 12.5, 15, 22.5 and 25 degrees respectively. This means an increased value about 55% for relative energy loss, when the classical jump is compared with a jump in a 25 degrees divergence angle.
Figure 2: Comparison of the relative sequent depths in rectangular cross section and the expanding channel with different divergence angle: (a) $\Theta = 5^\circ$, (b) $\Theta = 12.5^\circ$, (c) $\Theta = 15^\circ$, (d) $\Theta = 22.5^\circ$, (e) $\Theta = 25^\circ$, (f) all data

The length of jump was another considered hydraulic characteristic and was only measured for the constant rectangular cross section (equal to width of the flume) and the expanding channels with the divergence angles of 5, 12.5 and 15 degrees. The jump length was not measured for the other two divergence angles due to the very short length of these diversion parts. The ratios $1/y_L$ for all type of channels were calculated according to the measured values and are shown against the corresponding measured Froude numbers in Figure 4. As can be seen from this figure the length of jump is significantly decreased by increasing the divergence angle. For example, for the Froude numbers of 2 and 4 the ratios of $L_j/y_1$ were calculated as $(10.0, 7.5, 3.5, 3.0)$ and $(28.0, 15.0, 7.0, 5.8)$ for the classical jump and expanding channels with the divergence angles of 5, 12.5, 15 degrees respectively. The average reductions of this ratio for this range of Froude number are 35.5%, 70% and 74.5% for the divergence angles of 5, 12.5 and 15 degrees respectively.
Figure 3: Comparison of the relative energy loss in rectangular cross section and the expanding channel with different divergence angle: (a) $\Theta=5^\circ$, (b) $\Theta=12.5^\circ$, (c) $\Theta=15^\circ$, (d) $\Theta=22.5^\circ$, (e) $\Theta=25^\circ$, (f) all data

CONCLUSIONS

Extensive experiments were carried out to evaluate the hydraulic characteristics of jump in the expanding channels (i.e. transitions) with different
divergence angles. Five divergence angles, $5^\circ$-$25^\circ$ were considered and the obtained results were compared with each other and with the corresponding values for the classical jump. The main conclusions from this research study can be drawn as:

1. The sequent depths ratio was generally decreased with increasing the divergence angles. However, for the angles more than 15 degrees there was not significant effect on this ratio. For the Froude number range 2 to 4 the average reduction in this ratio for the angle of 25 degrees was about 27%.

2. The relative energy loss was increased with increasing the divergence angles. The results showed that for the angles more than 15 degrees, considerable changes were not observed between the relative energy losses with increasing this angle for the same conditions. As an example the relative energy loss in a 25 degrees divergent angle was obtained 55% greater than the corresponding value for a classical jump (Froude number equal 4).

3. It was found that the jump length can be considerably decreased in an expanding channel in comparison with the classical jump. For instance at Froude number 4 and divergence angle of 12.5 degrees the relative jump length ($L_j / y_1$) decreased by about 70%.

REFERENCES


